This paper explains the research conducted by Minitab statisticians to develop the methods and data checks used in the Assistant in Minitab Statistical Software.

Gage R&R Study (Crossed)

Overview

Measurement system studies are performed in virtually every type of manufacturing industry to properly monitor and improve a production process. In a typical measurement system study, a gage is used to obtain repeated measurements on selected parts by several operators. Two components of measurement system variability are frequently generated in such studies: repeatability and reproducibility. Repeatability represents the variability when the gage is used to measure the same part by the same operator. Reproducibility refers to the variability from different operators measuring the same part. Thus, measurement system studies are often referred to as gage repeatability and reproducibility studies, or gage R&R studies.

The primary purpose of a gage study is to determine how much variation in the data is due to the measurement system, and whether the measurement system is capable of assessing process performance. For detailed discussions on measurement system studies, refer to the MSA manual (2003), Montgomery and Runger (1993), and Burdick, Borror, and Montgomery (2005).

The Gage R&R Study (Crossed) command in the Assistant is designed to analyze data from typical measurement system studies. It adopts the most common approach of fitting the measurement data with an ANOVA model and estimates different sources of variation in the measurement system using the variance components in the model.

If you use the typical guidelines for how much data to collect for gage R&R studies, the variance components may not be precisely estimated (Montgomery and Runger, 1993a, 1993b; Vardeman and VanValkenburg, 1999). The Assistant indicates whether the number of parts and the number of operators are less than certain values, which may affect the precision of the part-to-part and operator variation estimates. We conducted simulations to identify the number of parts, operators, and replicates that are needed to obtain precise estimates.

Using our simulation results and widely accepted practices in measurement system analysis, we developed the following data checks for Gage R&R Study (Crossed). The Assistant automatically performs these data checks and reports the findings in the Report Card.

- Amount of Data
    - Process variation
    - Measurement variation

In this paper, we investigate how these data checks relate to measurement system analysis in practice and we describe how we established the guidelines for each data check.

Data checks

Amount of data

Typically, guidelines for gage R&R studies recommend using 10 parts, 2 or 3 operators, and 2 or 3 replicates (AIAG, 2003; Raffaldi and Ramsier, 2000; Tsai, 1988). However, the recommended sample size is not large enough to estimate part-to-part variation with good precision and, therefore, may not provide a good basis for assessing whether or not to use a particular gage (Montgomery and Runger, 1993a, 1993b; Vardeman and VanValkenburg, 1999).

To establish guidelines for the appropriate amount of data, we focused on how many parts should be evaluated to obtain estimates of part-to-part variation with different levels of precision. We also evaluated how many operators should be used to obtain a precise estimate of measurement variation. Finally, we investigated the number of observations required to obtain gage repeatability estimates with different precisions.

Number of parts to estimate part-to-part variation with different levels of precision

Objective

We wanted to determine how many parts should be evaluated to obtain estimates of part-to-part variation with different levels of precision.

Method

We performed a simulation study using 5000 samples. For all samples, we estimated the standard deviation of the parts and calculated the ratio of the estimated standard deviation to the true standard deviation. We sorted the ratios from low to high and used the 125^{th} and 4875^{th} ratios to define the 95% confidence interval; the 250^{th} and 4750^{th} ratios define the 90% confidence interval. Using these confidence intervals, we identified how many parts are needed to estimate part-to-part variation with different levels of precision.

Results

Based on the simulation study, we concluded the following:

- Using 10 parts, 3 operators, and 2 replicates, the ratio of the 90% confidence interval over the true standard deviation is about (0.61, 1.37) with 35% to 40% margin of error. At 95% confidence, the interval is about (0.55, 1.45) with 45% margin of error. Therefore, 10 parts are not enough to produce a precise estimate for the part-to-part variation component.

- You need approximately 35 parts to have a 90% confidence of estimating the part-to-part variation within 20% of the true value.
- You need approximately 135 parts to have a 90% confidence of estimating the part-to-part variation within 10% of the true value.

We also determined that these results apply to acceptable, marginal, and unacceptable gages.

See Appendix A for a detailed explanation of the simulation and its results.

Number of operators to estimate part-to-part variation with different levels of precision

Objective

We wanted to determine how many operators should evaluate parts to obtain operator variation estimates with different levels of precision.

Method

The standard deviation for parts and the standard deviation for operators are both estimated using the ANOVA model. Therefore, the method used in the simulation for the number of parts to estimate part-to-part variation also applies to the number of operators to estimate the variation between operators.

Results

Two or three operators are not enough to provide a precise estimate for reproducibility. However, the problem is less critical if the magnitude of part-to-part variation is much larger than the variation among operators, which is a likely scenario for many applications.

See Appendix A for a detailed explanation of the simulation and its results.

Number of observations to estimate repeatability with different levels of precision

Objective

We wanted to determine how the number of observations affects the estimate of repeatability and whether 10 parts, 3 operators, and 2 replicates can provide a reasonably precise estimate for repeatability variation.

Method

The ratio of the estimated repeatability standard deviation over its true value follows a chi-square distribution. To determine the number of observations needed to obtain a reasonably precise estimate of repeatability, we calculated the lower and upper bounds of the ratio associated with 90% probability and graphed the results.

Results

In a typical gage study (for example, number of parts = 10, number of operators = 3, and number of replicates = 2), the degrees of freedom for error equals 30, which allows you to have about 90% confidence of estimating the repeatability within 20% of the true value. Under typical settings, the estimate for repeatability is reasonably precise. See Appendix B for more details.

Overall results

Our studies clearly indicate that the typical settings used in a gage study are not good enough to provide precise estimates for part-to-part variation and reproducibility variation, which affect the ratio of the gage variation over the total process variation, and ultimately the decision about whether the gage is acceptable. Typically, part-to-part variation is greater than reproducibility variation, and therefore its precision has a greater impact on whether to accept a gage. However, in many applications, it may not be feasible to select 35 or more parts and have multiple operators measure them twice.

Considering the typical gage R&R settings used in practice and our simulation results, the Assistant uses the following approaches to encourage users to obtain precise estimates for the variance components:

1. Provide an option in the dialog box to allow users to enter an estimate of process variation obtained from a large historical data set. In most cases, the estimate from a large historical data set has better precision than the estimate from the sample data.
2. If the historical estimate is not available, and the number of parts is small, we display a message to remind users to select more than 10 parts to obtain more precise estimates.

Based on the amount of data, the Assistant Report Card displays information about process variation and measurement variation. For example, if you use 10 parts and 3 operators and specify a historical standard deviation, the following data check is displayed in the Report Card:

Status	Condition
i	To determine if a measurement system is capable of assessing process performance, you need good estimates of the process variation and the measurement variation. Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You entered a historical standard deviation so both estimates are available. You can compare them to see how well they agree. Although the number of parts in this study (10) satisfies the typical requirement of 10, the historical value should provide a more precise estimate of the process variation. Measurement variation: Estimated from the parts, it is broken down into Reproducibility and Repeatability. The number of parts (10) and operators (3) meets the typical requirement of 10 parts and 3 operators. This is usually adequate for estimating Repeatability, but the estimate of Reproducibility is less precise. If the %Process for Reproducibility estimate is large, you may want to examine the differences between operators and determine if these differences are likely to extend to other operators.

Below are all the messages for various configurations of parts, operators, and replicates.

PROCESS VARIATION

Historical standard deviation (parts < 10)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You entered a historical standard deviation so both estimates are available. You can compare them to see how well they agree. Because the number of parts in this study is small, the historical value should provide a more precise estimate of the process variation.

Historical standard deviation (parts ≥ 10, ≤ 15)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You entered a historical standard deviation so both estimates are available. You can compare them to see how well they agree. Although the number of parts in this study satisfies the typical requirement of 10, the historical value should provide a more precise estimate of the process variation.

Historical standard deviation (parts > 15, < 35)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You entered a historical standard deviation so both estimates are available. You can compare them to see how well they agree. The number of parts in this study is much larger than the typical requirement of 10. If the selected parts represent typical process variability, this estimate of the process variation should be much better than if you used 10 parts.

Historical standard deviation (parts ≥ 35)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You entered a historical standard deviation so both estimates are available. You can compare them to see how well they agree. The number of parts in this study is much larger than the typical requirement of 10. If the selected parts represent typical process variability, this estimate of the process variation should be adequate.

No historical standard deviation (parts < 10)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You chose to estimate from the parts but have fewer than the typical requirement of 10. The precision of this estimate may not be adequate. If the selected parts do not represent typical process variability, consider entering a historical estimate or using more parts.

No historical standard deviation (parts ≥ 10, ≤ 15)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You chose to estimate from the parts. Although the number of parts satisfies the typical requirement of 10, the estimate may not be precise. If the selected parts do not represent typical process variability, consider entering a historical estimate or using more parts.

No historical standard deviation (parts > 15, < 35)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You chose to estimate from the parts. The number of parts is much larger than the typical requirement of 10. If the selected parts represent typical process variability, this estimate of the process variation should be much better than if you used 10 parts

No historical standard deviation (parts ≥ 35)

- Process variation: Comprised of part-to-part and measurement variation. It can be estimated from a large sample of historical data, or from the parts in the study. You chose to estimate from the parts. The number of parts is much larger than the typical requirement of 10. If the selected parts represent typical process variability, this estimate of the process variation should be adequate.

MEASUREMENT VARIATION

Operators ≤ 2 or Parts < 10

- Measurement variation: Estimated from the parts, it is broken down into Reproducibility and Repeatability. The number of parts and operators does not meet the typical requirement of 10 parts and 3 operators. The estimates of measurement variation may not be precise. You should view the estimates as indicating general tendencies, rather than precise results.

Operators ≥ 3 and ≤ 5 and parts ≥ 10

- Measurement variation: Estimated from the parts, it is broken down into Reproducibility and Repeatability. The number of parts or operators meets the typical requirement of 10 parts and 3 operators. This is usually adequate for estimating Repeatability, but the estimate of Reproducibility is less precise. If the %Process for Reproducibility estimate is large, you may want to examine the differences between operators and determine if these differences are likely to extend to other operators.

Operators > 5 and parts ≥ 10

- Measurement variation: Estimated from the parts, it is broken down into Reproducibility and Repeatability. The number of parts or operators meets the typical requirement of 10 parts and 3 operators, and is usually adequate for estimating Repeatability. The additional operators improve the precision of the Reproducibility estimate.

References

Burdick, R.K., Borror, C. M., and Montgomery, D.C. (2005). *Design and analysis of gauge R&R studies: Making decisions with confidence intervals in random and mixed ANOVA models*. Philadelphia, PA: Society for Industrial Applied Mathematics (SIAM).

Automotive Industry Action Group (AIAG) (2003). *Measurement systems analysis (MSA) manual* (*3rd edition*). Southfield, MI: Chrysler, Ford, General Motors Supplier Quality Requirements Task Force.

Montgomery, D.C. (2000). *Design and analysis of experiments*. New York, NY: Wiley.

Montgomery, D.C., and Runger, G.C. (1993 a). Gage capability and designed experiments. Part I: Basic methods. *Quality Engineering*, *6 (1993/1994)*, *115 – 135*.

Montgomery, D.C., and Runger, G.C. (1993 b). Gage capability analysis and designed experiments. Part II: Experimental design models and variance component estimation. *Quality Engineering, 6 (1993/1994)*, *289-305*.

Raffaldi, J. and Ramsier, S. (2000). 5 ways to verify your gages. *Quality Magazine*, *39 (3)*, *38-42*.

Tsai, P. (1988). Variable gage repeatability and reproducibility study using the analysis of variance method. *Quality Engineering, 1(1)*, *107-115*.

Vardeman, S.B. and VanValkenburg, E.S. (1999). Two-way random-effects analyses and gage R&R studies. *Technometrics, 41 (3)*, *202-211*.

Appendix A: Evaluate the effect of parts on part-to-part variation

Because there is no exact formula to calculate the confidence interval for the part-to-part standard deviation, we performed a simulation to estimate the interval. To focus the simulation on how the number of parts affects the precision of the estimated part-to-part variation, we examined the ratio of the estimated confidence interval for the standard deviation of the parts over the true standard deviation of the parts. As the number of parts increases, the interval becomes narrower. We then identified the number of parts such that the margin of error for the ratio is 10% or 20%. The interval for the 10% margin of error is (0.9, 1.1), and for the 20% margin of error is (0.8, 1.2).

Simulation setup

A gage R&R study assumes that the k^{th} measurement of the i^{th} part by the j^{th} operator, denoted as Y_{ijk}, fits the following model:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}$$

Where

$i = 1, \dots, I, j = 1, \dots, J, k = 1, \dots, K$, and

α_i, β_j, γ_{ij}, and ε_{ijk} are independently normally distributed with mean 0, and variances of σ_P^2, σ_O^2, σ_{OP}^2, and σ_e^2. Here α_i, β_j, γ_{ij}, and ε_{ijk} represent parts, operators, parts x operators, and error terms.

Let r be the ratio of the total gage standard deviation over the total process standard deviation. Then,

$$r = \frac{\sqrt{RepeatabilityVariance + ReproducibilityVariance}}{\sqrt{PartsVariance + RepeatabilityVariance + ReproducibilityVariance}} = \frac{\sqrt{\sigma_e^2 + \sigma_O^2 + \sigma_{PO}^2}}{\sqrt{\sigma_P^2 + \sigma_e^2 + \sigma_O^2 + \sigma_{PO}^2}}$$

Typically, the following rule is used to determine whether a measurement system is acceptable:

r ≤ 0.1 (10%): acceptable

0.1 < r ≤ 0.3: marginal

0.3 < r: unacceptable

We choose r = 0.1 (acceptable), r = 0.25 (marginal), and r = 0.35 (unacceptable) to define the three regions. For the purposes of the simulation, we assume that the repeatability variance equals the reproducibility variance, which gives:

$$\frac{\sqrt{\sigma_e^2 + \sigma_e^2}}{\sqrt{\sigma_P^2 + 2\sigma_e^2}} = r \Rightarrow \sigma_P = \frac{\sqrt{(2 - 2r^2)}}{r}\sigma_e$$

We use $\sigma_{e=}0.001$ and 1, $\sigma_O^2 = \sigma_{PO}^2 = 0.5\sigma_e^2$, and $\sigma_P = \frac{\sqrt{(2-2r^2)}}{r}\sigma_e$ to generate the observations, and assume that 3 operators measure each part twice to evaluate how the number of parts affects the standard deviation of the parts.

These are the simulation steps we followed for each number of parts, r, and σ_e:

1. Generate 5000 samples using the model above.
2. Estimate part standard deviation, and calculate the ratio of the estimated standard deviation over the true standard deviation for all 5000 samples.
3. Sort the 5000 ratios in increasing order. Of the 5000 sorted ratios, the 125th and 4875th ratios represent the lower and upper bounds of the interval at the 95% confidence level, and the 250th and 4750th ratios represent the lower and upper bounds of the interval at the 90% confidence level.
4. Examine the intervals to identify the number of parts such that the margin of error is 10% or 20%. The interval for the 10% margin of error is (0.9, 1.1). The interval for the 20% margin of error is (0.8, 1.2).

Simulation results

The results in Tables 1-6 show the simulation results at each confidence level for different numbers of parts, with each table corresponding to a specific combination of values for r and σ_e. Overall, these results show that:

- Using 10 parts, 3 operators, and 2 replicates, the ratio of the 90% confidence interval over the true standard deviation is about (0.61, 1.37) with 35% to 40% margin of error. At the 95% confidence level, the interval is about (0.55, 1.45) with 45% margin of error. Therefore, 10 parts are not enough to produce a precise estimate for the part-to-part variation component.
- You need approximately 35 parts to have a 90% confidence of estimating the part-to-part variation within 20% of the true value.
- You need approximately 135 parts to have a 90% confidence of estimating the part-to-part variation within 10% of the true value.

Note that this summary of the results is not specific to a particular combination of r and σ_e. The rows corresponding to the bulleted results above are highlighted in Tables 1, 2, 3, 4, 5, and 6 below.

Table 1 Acceptable gage (r = 0.1), $\sigma_e = 0.001$, true part stdev = 0.014071247

	Ratio of estimated confidence interval for part stdev/true part stdev	
Number of parts	95% Confidence	90% Confidence
3	(0.15295, 1.93755)	(0.22195, 1.73365)
5	(0.34415, 1.67035)	(0.41861, 1.53873)
10	(0.55003, 1.44244)	(0.60944, 1.36992)
15	(0.63295, 1.36927)	(0.68721, 1.30294)
20	(0.68532, 1.31187)	(0.72950, 1.25701)
25	(0.71230, 1.27621)	(0.75578, 1.23251)
30	(0.74135, 1.24229)	(0.77645, 1.20841)
35	(0.76543, 1.23033)	(0.80066, 1.19706)
50	(0.79544, 1.20337)	(0.82636, 1.16595)
100	(0.85528, 1.13696)	(0.88063, 1.11635)
135	(0.87686, 1.12093)	(0.89448, 1.09760)
140	(0.88241, 1.11884)	(0.90130, 1.09974)

Table 2 Acceptable gage (r =0.1), $\sigma_e = 1$, true part stdev = 14.071247

	Ratio of estimated confidence interval for part stdev/true part stdev	
Number of parts	95% Confidence	90% Confidence
5	(0.34656, 1.68211)	(0.42315, 1.55880)
10	(0.55496, 1.45382)	(0.61319, 1.38233)
15	(0.63484, 1.36949)	(0.68767, 1.30505)
35	(0.76233, 1.23513)	(0.79749, 1.19623)
40	(0.77256, 1.21518)	(0.81224, 1.18121)
135	(0.88017, 1.12345)	(0.89883, 1.10249)
140	(0.88004, 1.11725)	(0.89787, 1.09713)

	Ratio of estimated confidence interval for part stdev/true part stdev	
Number of parts	95% Confidence	90% Confidence
145	(0.88281, 1.11886)	(0.89966, 1.09583)
150	(0.88302, 1.11132)	(0.90096, 1.09296)

Table 3 Marginal gage (r = 0.25), $\sigma_e = 0.001$, true part stdev = 0.005477225575

	Ratio of estimated confidence interval for part stdev/true part stdev	
Number of parts	95% Confidence	90% Confidence
30	(0.73879, 1.25294)	(0.77982, 1.21041)
35	(0.75881, 1.24383)	(0.79848, 1.20068)
40	(0.77281, 1.22813)	(0.80369, 1.18788)
135	(0.87588, 1.11910)	(0.89556, 1.10093)
140	(0.87998, 1.12001)	(0.89917, 1.09717)
145	(0.88100, 1.11812)	(0.89852, 1.09710)
150	(0.88373, 1.11563)	(0.90345, 1.09706)

Table 4 Marginal gage (r = 0.25), $\sigma_e = 1$, true part stdev = 5.477225575

	Ratio of estimated confidence interval for part stdev/true part stdev	
Number of parts	95% Confidence	90% Confidence
30	(0.74292, 1.25306)	(0.78159, 1.20872)
35	(0.76441, 1.24391)	(0.79802, 1.20135)
40	(0.77525, 1.21339)	(0.80786, 1.17908)
135	(0.87501, 1.11711)	(0.89512, 1.09758)
140	(0.87934, 1.11756)	(0.89881, 1.09862)
145	(0.88308, 1.11530)	(0.90056, 1.09806)

Table 5 Unacceptable gage (r = 0.35), $\sigma_e = 0.001$, true part stdev = 0.00378504

	Ratio of estimated confidence interval for part stdev/true part stdev	
Number of parts	95% Confidence	90% Confidence
30	(0.74313, 1.25135)	(0.77427, 1.20568)
35	(0.75409, 1.24332)	(0.79444, 1.19855)
40	(0.76582, 1.22289)	(0.80599, 1.18615)
135	(0.87641, 1.12043)	(0.89507, 1.09820)
140	(0.87635, 1.11539)	(0.89651, 1.09368)
145	(0.88339, 1.11815)	(0.89772, 1.09591)

Table 6 Unacceptable gage (r = 0.35), $\sigma_e = 1$, true part stdev = 3.78504

	Ratio of estimated confidence interval for part stdev/true part stdev	
Number of parts	95% Confidence	90% Confidence
30	(0.73750, 1.26100)	(0.77218, 1.21285)
35	(0.74987, 1.23085)	(0.79067, 1.18860)
40	(0.77187, 1.22270)	(0.80648, 1.18329)
135	(0.87572, 1.11877)	(0.89409, 1.09827)
140	(0.87798, 1.11634)	(0.89590, 1.09695)
145	(0.87998, 1.11513)	(0.89683, 1.09534)

Number of operators

The standard deviation for parts and the standard deviation for operators are estimated identically using the ANOVA model. Therefore, the simulation results on parts also apply to reproducibility variation. Two or three operators are not enough to provide a precise estimate for reproducibility. However, the problem is less critical for operators if the magnitude of part-to-part variation is much larger than the operator variation, which is a likely scenario for many applications.

For example, suppose part-to-part standard deviation is 20 times the operator standard deviation. The part standard deviation is 20, and operator standard deviation is 1. Assuming repeatability is the same as reproducibility, then the true ratio of measurement system variation over the total process variation is:

$$\sqrt{\frac{1+1}{400+1+1}} = 0.0705$$

Now assume the margin of error for estimating the operator standard deviation is 40% (high). That is, the estimated operator standard deviation could be 1.4. Therefore, the ratio of the measurement system overall the total becomes:

$$\sqrt{\frac{1.4^2+1.4^2}{400+1.4^2+1.4^2}} = 0.0985$$

Because this value is less than 0.10, a large reproducibility variation does not affect gage acceptance if 10% is the cutoff value.

If the operator variation is nearly the same as part variation, you need a large number of operators to represent the measurement system and to accurately evaluate the gage.

Appendix B: Estimating repeatability

Calculation setup

Unlike confidence intervals for the part-to-part standard deviation, which are based on an approximation, the ratio of the estimated repeatability standard deviation over its true value follows a chi-square distribution. Therefore, we can calculate the lower and upper bounds of the ratio associated with 90% probability, and then evaluate how both bounds approach 1 as the number of parts, number of operators, and the number of replicates increase.

Using the same notation defined in Appendix A, the repeatability variance is estimated by

$$S^2 = \sum (Y_{ijk} - \bar{Y}_{ij.})^2 / IJ(K-1)$$

Then, $\dfrac{IJ(K-1)S^2}{\sigma_e{}^2}$ follows a chi-square distribution with IJ(K-1) degrees of freedom (df), where *I* is the number of parts, *J* is the number of operators, and *K* is the number of replicates.

Based on this result, the ratio of the estimated standard deviation over its true value satisfies the following probability equation:

$$Probability\left(\sqrt{\frac{\chi^2_{df,\alpha/2.}}{df}} \le \frac{S}{\sigma_e} \le \sqrt{\frac{\chi^2_{df,1-(\alpha/2)}}{df}}\right) = 1-\alpha$$

where df = IJ(K-1) = number of parts * number of operators * (number of replicates – 1). If the number of replicates equals 2, the degrees of freedom equal the number of parts times the number of operators.

Using this formula, for each given value of the degrees of freedom, we calculate the lower and upper bounds of the ratio $\dfrac{S}{\sigma_e}$ at a probability of 90%. We then identify the degrees of freedom such that the estimated standard deviation is within 10% and 20% of its true value. The corresponding interval is (0.9, 1.1) for the 10% margin of error, and (0.8, 1.2) for the 20% margin of error.

Calculation results

The graph in Figure 1 shows the lower and upper bounds of the ratio $\dfrac{S}{\sigma_e}$ at 90% probability versus the degrees of freedom, with the degrees of freedom ranging from 1 to 200.

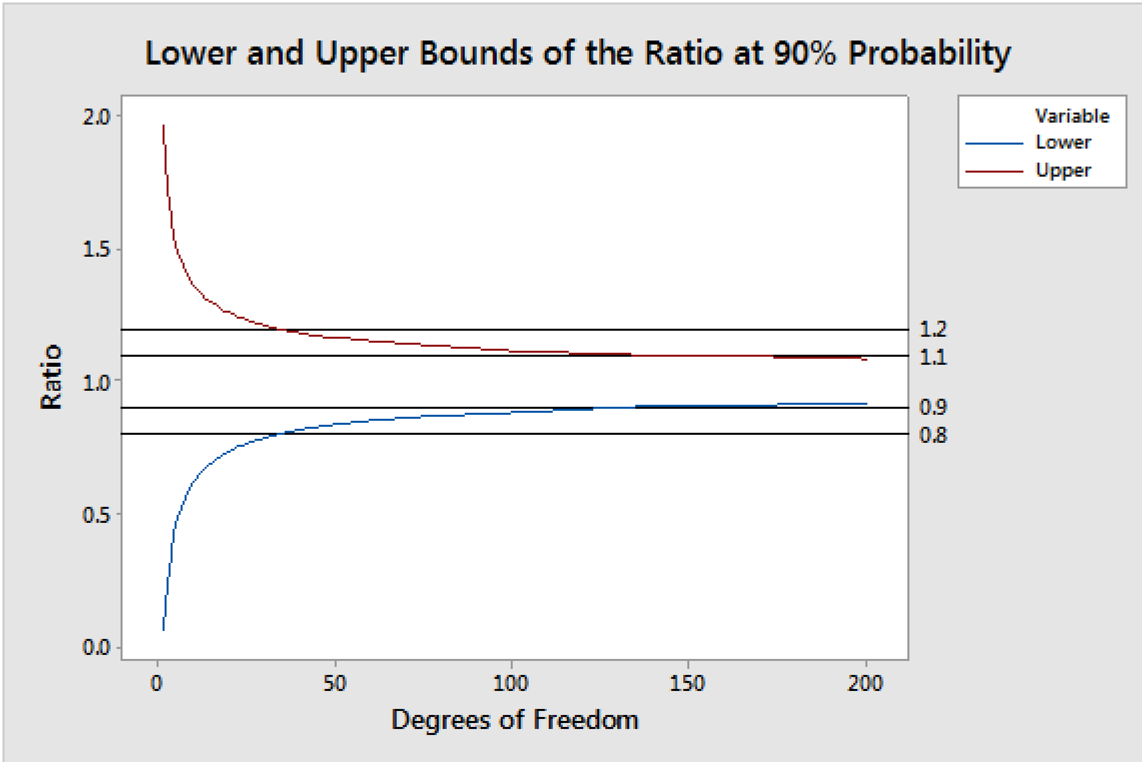

Figure 1 Lower and upper bounds of $\frac{S}{\sigma_e}$ at 90% probability versus degrees of freedom (1 to 200)

Notice that that the interval formed by the lower and upper bounds narrows as the degrees of freedom increase. The width of the interval decreases dramatically as the degrees of freedom increase from 1 to 50. We can see this more clearly in the enlarged graph shown in Figure 2, which displays the results for degrees of freedom from 1 to 50.

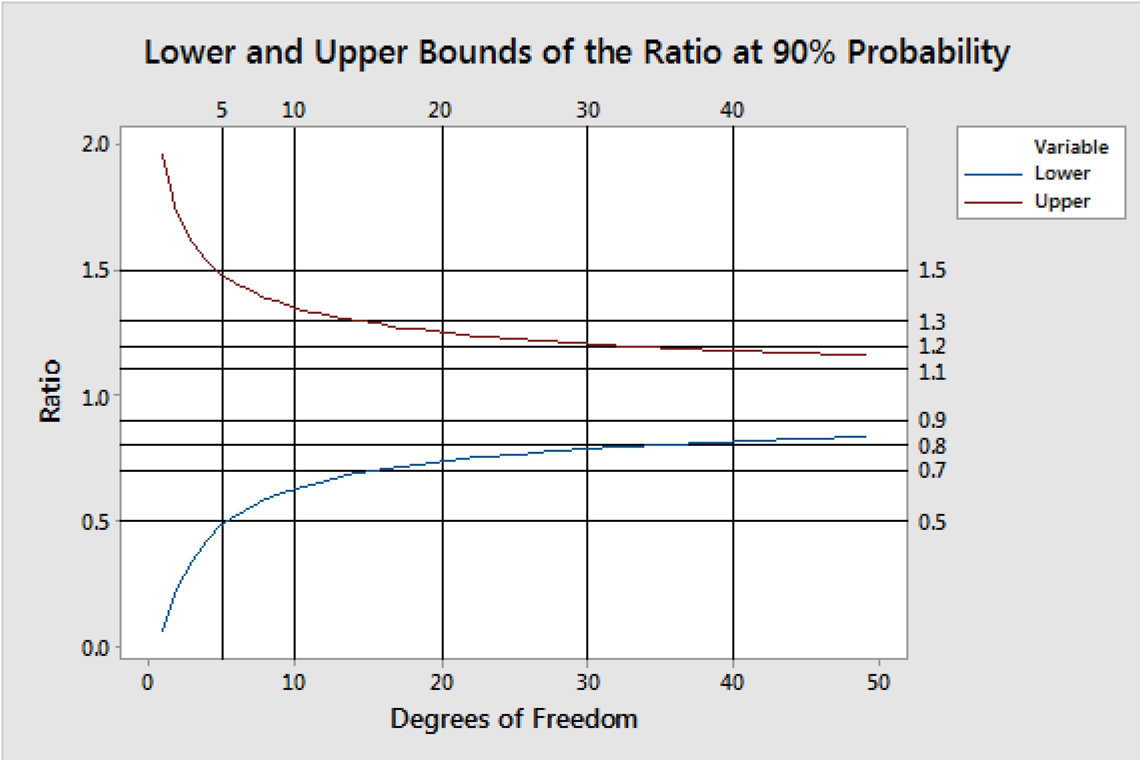

Figure 2 Lower and upper bounds of $\frac{S}{\sigma_e}$ at 90% probability versus degrees of freedom (1 to 50)

As shown in Figure 2, when the degrees of freedom are less than 10, the interval is wider than (0.63, 1.35). As the degrees of freedom increase, the interval becomes narrower, as indicated by the values in Table 7 below.

Table 7 Degrees of freedom and lower and upper bounds at 90% probability

Degrees of freedom	Interval formed by lower and upper bounds
5	(0.48,1.49)
10	(0.63, 1.35)
15	(0.70, 1.29)
20	(0.74, 1.25)
25	(0.76, 1.23)
30	(0.79, 1.21)

Degrees of freedom	Interval formed by lower and upper bounds
35	(0.80, 1.19)
40	(0.81, 1.18)

Therefore, at 90% probability, you need about 35 degrees of freedom to obtain a 20% margin of error for the standard deviation estimate of repeatability. Recall that the degrees of freedom equal the Number of Parts * Number of Operators * (Number of Replicates – 1). Therefore, the typical recommendation of 10 parts, 3 operators, and 2 replicates provides degrees of freedom (30) that are close to this requirement. To obtain a 10% margin of error at 90% probability, you need about 135 degrees of freedom (see Figure 1).